Holographic Dark Energy in a Non-flat Universe with Granda-Oliveros Cut-off

K. Karami · J. Fehri

Abstract Motivated by Granda and Oliveros (GO) model, we generalize their work to the non-flat case. We obtain the evolution of the dark energy density, the deceleration and the equation of state parameters for the holographic dark energy model in a non-flat universe with GO cut-off. In the limiting case of a flat universe, i.e. $k = 0$, all results given in GO model are obtained.

Keywords Holography · Dark energy · Redshift

1 Introduction

Type Ia supernovae observational data suggest that our universe is experiencing an accelerated expansion driven by an exotic energy with negative pressure which is so-called dark energy (DE) [1–4]. However, the nature of DE is still unknown, and people have proposed some candidates to describe it. The cosmological constant, Λ, is the most obvious theoretical candidate of DE, which has the equation of state $\omega = -1$. Astronomical observations indicate that the cosmological constant is many orders of magnitude smaller than estimated in modern theories of elementary particles [5]. Also the "fine-tuning" and the "cosmic coincidence" problems are the two well-known difficulties of the cosmological constant problems [6].

There are different alternative theories for the dynamical DE scenario which have been proposed by people to interpret the accelerating universe. (i) The scalar field models of DE including quintessence [7, 8], phantom (ghost) field [9–11], K-essence [12–14] based on earlier work of K-inflation [15, 16], tachyon field [17–23], dilatonic ghost condensate [24–26], quintom [27–29], and so forth. (ii) The DE models including Chaplygin gas [30,

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31], braneworld models [32, 33], agegraphic DE models [34, 35], and $f(R)$-gravity models [36–44], etc.

Recently, a new DE candidate, based on the holographic principle, was proposed [45–47]. According to the holographic principle, the number of degrees of freedom in a bounded system should be finite and has relations with the area of its boundary [48, 49]. In quantum field theory a short distance (UV) cut-off Λ is related to a long distance (IR) cut-off L due to the limit set by formation of a black hole, which results in an upper bound on the zero-point energy density [50]. By applying the holographic principle to cosmology, one can obtain the upper bound of the entropy contained in the universe [51]. Following this line, Li [52] argued that for a system with size L and UV cut-off Λ, it is required that the total energy in a region of size L should not exceed the mass of a black hole of the same size, thus $L^3\rho_\Lambda \leq LM_P^2$, where ρ_Λ is the quantum zero-point energy density caused by UV cut-off Λ and M_P is the reduced Planck Mass $M_P^{-2} = 8\pi G$. The largest L allowed is the one saturating this inequality, thus $\rho_\Lambda = 3c^2M_P^2L^{-2}$, where c is a numerical constant. Recent observational data, which have been used to constrain the holographic DE model, show that for the non-flat universe $c = 0.815^{+0.179}_{-0.139}$ [53], and for the flat case $c = 0.818^{+0.113}_{-0.097}$ [54]. Also Li [52] showed that the cosmic coincidence problem can be resolved by inflation in the holographic DE model, provided the minimal number of e-foldings [52]. The holographic models of DE have been studied widely in the literature [55–75]. As we mentioned before, the UV cut-off is related to the vacuum energy, and IR cut-off is related to the large scale of the universe, for example Hubble horizon, future event horizon or particle horizon. Taking L as the size of the current universe, for instance, the Hubble scale, the resulting energy density is comparable to the present day DE. However, as found by Hsu [76], in that case, the evolution of the DE is the same as that of dark matter (dust matter), and therefore it cannot drive the universe to accelerated expansion. The same appears if one chooses the particle horizon of the universe as the length scale L [52]. An interesting proposal is made by Li [52]: Choosing the event horizon of the universe as the length scale, the holographic DE not only gives the observation value of DE in the universe, but also can drive the universe to an accelerated expansion phase. In that case, however, an obvious drawback concerning causality appears in this proposal. Event horizon is a global concept of space–time; existence of event horizon of the universe depends on future evolution of the universe; and event horizon exists only for universe with forever accelerated expansion. This motivated GO [77] to propose a new infrared cut-off for the holographic DE, which besides the square of the Hubble scale also contains the time derivative of the Hubble scale. This model depends on local quantities and avoids the problem of causality which appears using the event horizon area as the IR cut-off. They obtained the evolution of both the deceleration and the equation of state parameters for this model in a flat universe.

Besides, as usually believed, an early inflation era leads to a flat universe. This is not a necessary consequence if the number of e-foldings is not very large [78]. It is still possible that there is a contribution to the Friedmann equation from the spatial curvature when studying the late universe, though much smaller than other energy components according to observations. Therefore, it is not just of academic interest to study a universe with a spatial curvature marginally allowed by the inflation model as well as observations. Some experimental data have implied that our universe is not a perfectly flat universe and that it possesses a small positive curvature [79–83]. The tendency of preferring a closed universe appeared in a suite of CMB experiments [84–87]. The improved precision from WMAP provides further confidence, showing that a closed universe with positively curved space is marginally preferred [88–93]. In addition to CMB, recently the spatial geometry of the universe was probed by supernova measurements of the cubic correction to the luminosity distance [94, 95], where a closed universe is also marginally favored.

All mentioned in above motivate us to consider the holographic DE model with the new infrared cut-off proposed by [77] and extend their work to a non-flat case. To do this, in Sect. 2, we obtain the evolution of the DE density, the deceleration and the equation of state parameters for the holographic DE model given by [77] in the context of the non-flat universe. In Sect. 3, we give numerical results. Section 4 is devoted to conclusions.

2 Holographic DE in Non-flat FRW Universe with GO Cut-off

Following GO [77], the holographic DE density is given by

$$\rho_\Lambda = 3(\alpha H^2 + \beta \dot{H}), \tag{1}$$

where α and β are constants. GO [77] argued that since the underlying origin of the holographic DE is still unknown, the inclusion of the time derivative of the Hubble parameter may be expected as this term appears in the curvature scalar, and has the correct dimension. This kind of density may appear as the simplest case of more general $f(H, \dot{H})$ holographic density in the FRW background. Comparing (1) with the holographic DE density $\rho_\Lambda = 3c^2 M_P^2 L^{-2}$ shows that the corresponding IR cut-off L for the model (1) is

$$L = H^{-1}\left(1 + \frac{\beta}{\alpha}\frac{\dot{H}}{H^2}\right)^{-1/2}, \tag{2}$$

which depends on local quantities and avoids the problem of causality which appears using the event horizon area as the IR cut-off. For the special case $\beta = 0$, equation (2) yields the Hubble horizon as the IR cut-off, i.e. $L = H^{-1}$.

The first Friedmann equation in non-flat universe is given by

$$H^2 = \frac{1}{3}(\rho_m + \rho_r + \rho_\Lambda + \rho_k), \tag{3}$$

where we take $8\pi G = 1$ and $\rho_k = -3k/a^2$. Parameter k denotes the curvature of space $k = 0, 1, -1$ for a flat, closed and open universe, respectively. Let us define the current densities parameters for $a_0 = 1$ as usual

$$\Omega_{m_0} = \frac{\rho_{m_0}}{3H_0^2}, \quad \Omega_{r_0} = \frac{\rho_{r_0}}{3H_0^2}, \quad \Omega_{\Lambda_0} = \frac{\rho_{\Lambda_0}}{3H_0^2}, \quad \Omega_{k_0} = \frac{\rho_{k_0}}{3H_0^2} = -\frac{k}{H_0^2}, \tag{4}$$

where these densities satisfy $\Omega_{m_0} + \Omega_{r_0} + \Omega_{\Lambda_0} + \Omega_{k_0} = 1$. A closed universe with a small positive curvature ($\Omega_{k_0} \sim -0.015$) is compatible with observations [79–83].

Now we can rewrite the first Friedmann equation in terms of $x = \ln a$ as

$$\tilde{H}^2 = \Omega_{m_0}e^{-3x} + \Omega_{r_0}e^{-4x} + \Omega_{k_0}e^{-2x} + \alpha\tilde{H}^2 + \frac{\beta}{2}\frac{d\tilde{H}^2}{dx}, \tag{5}$$

where $\tilde{H} = \frac{H}{H_0}$ is the scaled Hubble expansion rate and H_0 is the present value of the Hubble parameter (for $x = 0$).

Solving (5), we obtain

$$\tilde{H}^2 = \frac{2}{3\beta - 2\alpha + 2}\Omega_{m_0}e^{-3x} + \frac{1}{2\beta - \alpha + 1}\Omega_{r_0}e^{-4x} + \frac{1}{\beta - \alpha + 1}\Omega_{k_0}e^{-2x} + Ce^{-2x(\alpha-1)/\beta}, \tag{6}$$

where C is an integration constant. From (1) and (6), the scaled holographic DE density, $\tilde{\rho}_\Lambda = \frac{\rho_\Lambda}{3H_0^2}$, can be obtained as

$$\tilde{\rho}_\Lambda = \frac{3\beta - 2\alpha}{2\alpha - 3\beta - 2}\Omega_{m_0}e^{-3x} + \frac{2\beta - \alpha}{\alpha - 2\beta - 1}\Omega_{r_0}e^{-4x} + \frac{\beta - \alpha}{\alpha - \beta - 1}\Omega_{k_0}e^{-2x} + Ce^{-2x(\alpha-1)/\beta}. \tag{7}$$

Using (6) and (7), the DE density parameter, $\Omega_\Lambda = \rho_\Lambda/3H^2 = \tilde{\rho}_\Lambda/\tilde{H}^2$, can be obtained in terms of redshift $z = \frac{1}{a} - 1$ as

$$\Omega_\Lambda = \frac{\frac{3\beta-2\alpha}{2\alpha-3\beta-2}\Omega_{m_0}(1+z)^3 + \frac{2\beta-\alpha}{\alpha-2\beta-1}\Omega_{r_0}(1+z)^4 + \frac{\beta-\alpha}{\alpha-\beta-1}\Omega_{k_0}(1+z)^2 + C(1+z)^{\frac{2(\alpha-1)}{\beta}}}{\frac{2}{3\beta-2\alpha+2}\Omega_{m_0}(1+z)^3 + \frac{1}{2\beta-\alpha+1}\Omega_{r_0}(1+z)^4 + \frac{1}{\beta-\alpha+1}\Omega_{k_0}(1+z)^2 + C(1+z)^{\frac{2(\alpha-1)}{\beta}}}. \tag{8}$$

Using (7) and the holographic DE conservation equation

$$\tilde{p}_\Lambda = -\tilde{\rho}_\Lambda - \frac{1}{3}\frac{d\tilde{\rho}_\Lambda}{dx}, \tag{9}$$

we obtain the DE pressure

$$\tilde{p}_\Lambda = \frac{2\alpha - 3\beta - 2}{3\beta}Ce^{-2x(\alpha-1)/\beta} + \frac{2\beta - \alpha}{3(\alpha - 2\beta - 1)}\Omega_{r_0}e^{-4x} + \frac{\beta - \alpha}{3(\beta - \alpha + 1)}\Omega_{k_0}e^{-2x}. \tag{10}$$

Note that for the special case $\beta = 0$, using (7) and (10), the equation of state (EoS) parameter of the DE, $\omega_\Lambda = \tilde{p}_\Lambda/\tilde{\rho}_\Lambda$, is obtained as

$$\omega_\Lambda = \frac{\Omega_{r_0}(1+z)^4 - \Omega_{k_0}(1+z)^2}{3[\Omega_{m_0}(1+z)^3 + \Omega_{r_0}(1+z)^4 + \Omega_{k_0}(1+z)^2]}, \tag{11}$$

which shows that neglecting the contributions from radiation and curvature, i.e. $\Omega_{r_0} = \Omega_{k_0} = 0$, yields the pressureless DE, i.e. $\omega_\Lambda = 0$, where its EoS behaves like the (dark matter) dust matter. This result is same as that obtained by Hsu [76] for the holographic DE model with the IR cut-off $L = H^{-1}$. Therefore choosing the Hubble horizon as the IR cut-off cannot drive the universe to accelerated expansion.

Using $\tilde{\rho}_k = \rho_k/3H_0^2 = \Omega_{k_0}a^{-2}$ and the continuity equation for the curvature

$$\tilde{p}_k = -\tilde{\rho}_k - \frac{1}{3}\frac{d\tilde{\rho}_k}{dx}, \tag{12}$$

we obtain the curvature pressure

$$\tilde{p}_k = -\frac{1}{3}\Omega_{k_0}a^{-2}. \tag{13}$$

By considering $\tilde{p}_{\Lambda_0} = \omega_0\tilde{\rho}_{\Lambda_0} = \omega_0\Omega_{\Lambda_0}$, one can obtain the following relations between the three constants α, β and C appeared in (7) and (10) as

$$C = 1 + \frac{2\Omega_{m_0}}{2(\Omega_{\Lambda_0} - 1) + \beta[3\Omega_{m_0} + 4\Omega_{r_0} + 3(1+\omega_0)\Omega_{\Lambda_0} + 2\Omega_{k_0} - 3]}$$

$$+ \frac{2\Omega_{r_0}}{2(\Omega_{\Lambda_0} - 1) + \beta[3\Omega_{m_0} + 4\Omega_{r_0} + 3(1+\omega_0)\Omega_{\Lambda_0} + 2\Omega_{k_0} - 4]}$$

$$+ \frac{2\Omega_{k_0}}{2(\Omega_{\Lambda_0} - 1) + \beta[3\Omega_{m_0} + 4\Omega_{r_0} + 3(1+\omega_0)\Omega_{\Lambda_0} + 2\Omega_{k_0} - 2]}, \tag{14}$$

and

$$\alpha = \frac{1}{2}\left[2\Omega_{\Lambda_0} + \beta\left(3\Omega_{m_0} + 4\Omega_{r_0} + 3(1+\omega_0)\Omega_{\Lambda_0} + 2\Omega_{k_0}\right)\right]. \tag{15}$$

The deceleration parameter is given by

$$q = \frac{1}{2} + \frac{3}{2}\left(\frac{\tilde{p}_r + \tilde{p}_\Lambda + \tilde{p}_k}{\tilde{\rho}_m + \tilde{\rho}_r + \tilde{\rho}_\Lambda + \tilde{\rho}_k}\right), \tag{16}$$

where $p_m = 0$ for dust matter, $\tilde{\rho}_m = \rho_m/3H_0^2 = \Omega_{m_0}a^{-3}$, $\tilde{\rho}_r = \rho_r/3H_0^2 = \Omega_{r_0}a^{-4}$, and $\tilde{p}_r = \frac{1}{3}\Omega_{r_0}a^{-4}$. In what follows we neglect the contribution from radiation, i.e. $\Omega_{r_0} = 0$.

Note that if one set $\Omega_{k_0} = 0$, then (6), (7), (10), (14) to (16) reduce to (2.5), (2.6), (2.8) to (2.11) in [77], respectively.
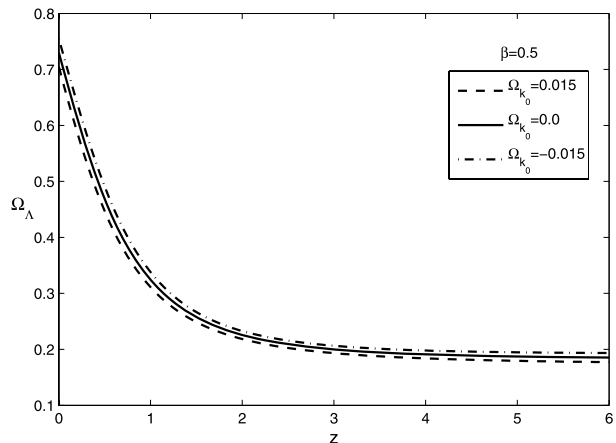
3 Numerical Results

The evolution of the DE density parameter Ω_Λ given by (8), the declaration parameter q given by (16) and the EoS parameter $\omega_\Lambda = \tilde{p}_\Lambda/\tilde{\rho}_\Lambda$, using (7) and (10), are displayed in Figs. 1–5. Figures 1 to 2 present the evolution of the DE density parameter Ω_Λ and the declaration parameter q versus redshift z for open ($\Omega_{k_0} = 0.015$), flat ($\Omega_{k_0} = 0.0$) and closed ($\Omega_{k_0} = -0.015$) universes with $\beta = 0.5$. Choosing a non-flat universe with $\Omega_{k_0} = -0.015$ is compatible with the recent observations [79–83]. Our numerical results in Figs. 1 to 2 show that in the presence of small curvature ($\Omega_{k_0} = \pm 0.015$), the average difference between the non-flat and the flat cases is order of 10^{-2}.
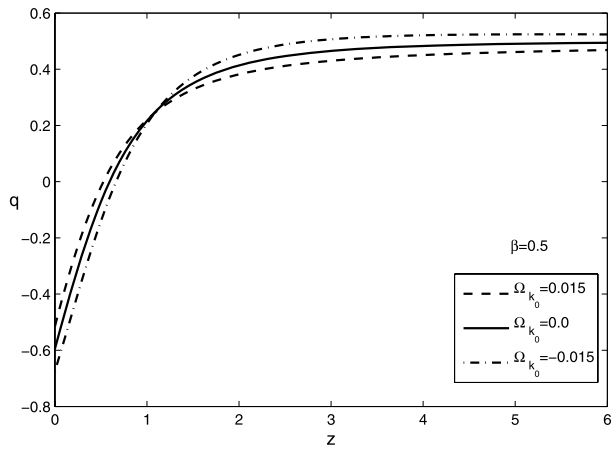
Figure 3 shows the evolution of the deceleration parameter versus redshift for closed ($\Omega_{k_0} = -0.015$) universe with the three different values of β. Note that Fig. 3 clears that for $\beta = 0.5$ and 0.7, the values of the transition redshift z_T same as the flat case ($\Omega_{k_0} = 0.0$) in [77], are consistent with the current observational data.

Figure 4 presents the evolution of the EoS parameter ω_Λ versus redshift z for open ($\Omega_{k_0} = 0.015$), flat ($\Omega_{k_0} = 0.0$) and closed ($\Omega_{k_0} = -0.015$) universes with $\beta = 0.5$. Figure 4 same as Fig. 2 shows an average difference $O(10^{-2})$ between the non-flat and the flat cases. Figure 4 shows that for the closed universe, the EoS parameter ω_Λ from nearly 0 at $z \simeq 4.4$ to -1 at $z \to 0$, same as the flat case in [77] and open universe with $\Omega_{k_0} = 0.015$, behaves like some
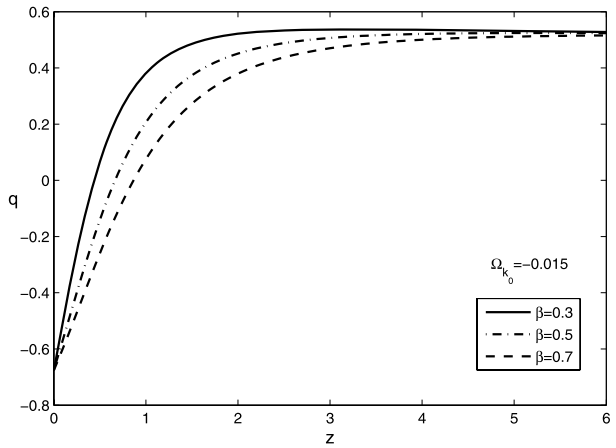

Fig. 1 DE density parameter versus redshift for open ($\Omega_{k_0} = 0.015$), flat ($\Omega_{k_0} = 0.0$) and closed ($\Omega_{k_0} = -0.015$) universes. Auxiliary parameters are: $w_0 = -1$, $\Omega_{m_0} = 0.27$, $\Omega_{r_0} = 0$, and $\beta = 0.5$
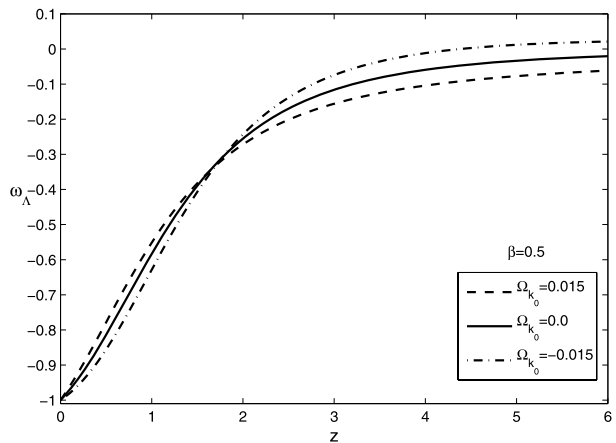
Fig. 2 Deceleration parameter versus redshift for open ($\Omega_{k_0} = 0.015$), flat ($\Omega_{k_0} = 0.0$) and closed ($\Omega_{k_0} = -0.015$) universes. Auxiliary parameters as in Fig. 1
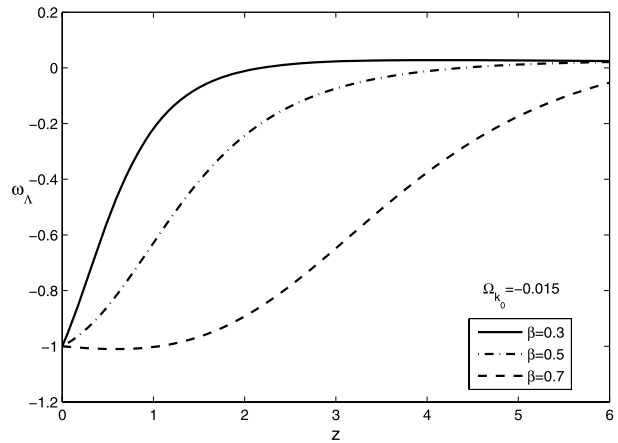

Fig. 3 Deceleration parameter versus redshift for closed universe with $\Omega_{k_0} = -0.015$ and for the different values of $\beta = 0.3$ (*solid line*), 0.5 (*dash-dotted line*) and 0.7 (*dashed line*). Auxiliary parameters are: $w_0 = -1$, $\Omega_{m_0} = 0.27$, and $\Omega_{r_0} = 0$

Fig. 4 EoS parameter versus redshift for open ($\Omega_{k_0} = 0.015$), flat ($\Omega_{k_0} = 0.0$) and closed ($\Omega_{k_0} = -0.015$) universes. Auxiliary parameters as in Fig. 1

Fig. 5 EoS parameter versus redshift for closed universe with $\Omega_{k_0} = -0.015$ and for the different values of $\beta = 0.3$ (*solid line*), 0.5 (*dash-dotted line*) and 0.7 (*dashed line*). Auxiliary parameters as in Fig. 3

scalar filed models of DE. Figure 5 shows the evolution of the EoS parameter versus redshift for closed ($\Omega_{k_0} = -0.015$) universe with the three different values of β.

4 Conclusions

We used a holographic DE model with new infrared cut-off proposed by GO [77], which includes a term proportional to $\dot{H}$. Contrary to the holographic DE based on the event horizon, this model depends on local quantities, avoiding in this way the causality problem, and solved the coincidence problem. Hence the proposed new infrared cut-off can be considered as a viable phenomenological model of holographic density. We extended GO model [77] to the non-flat case. However, some experimental data have implied that our universe is not a perfectly flat universe and that it possesses a small curvature ($\Omega_{k_0} \sim -0.015$) [79–83]. Although it is believed that our universe is flat, a contribution to the Friedmann equation from spatial curvature is still possible if the number of e-foldings is not very large [78]. We obtained the evolution of the DE density parameter Ω_Λ, the declaration parameter q and the EoS parameter ω_Λ for a non-flat universe. In the limiting case of a flat universe, the results are in exact agreement with those obtained by GO [77]. Our numerical results show that

(i) for the non-flat universe with small curvature ($\Omega_{k_0} = \pm 0.015$), the DE density, the declaration and the EoS parameters show an average difference $O(10^{-2})$ in comparison with the flat case;

(ii) for the closed universe with $\Omega_{k_0} = -0.015$ and for the declaration parameter with $\beta = 0.5$ and 0.7, the values of the transition redshift z_T same as the flat case are consistent with the current observational data;

(iii) for the closed universe with $\Omega_{k_0} = -0.015$, the EoS parameter ω_Λ for $0 \leq z < 4.4$, same as the flat and open ($\Omega_{k_0} = 0.015$) cases, behaves like some scalar filed models of DE.

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